Feature

Numbers Don't Lie – A Water Math Tutorial

By Matthew Wirth

Among people whose job it is to engineer systems and size equipment, there is a phrase commonly used by all: "You can't do that!" Playing with numbers and ignoring the laws of physics and mathematics will land one in a world of trouble—costing time, money and resources. There is a place for sound science and engineering and it is found in the numbers. While math was not a favorite subject for many students, it was an effective form of sedation. For those who slept through math class, it is not too late to learn—water math is crucial to effective water treatment. The good thing is the math whiz folks within the industry simplified many of the equations and developed handy charts and graphs to help the mathematically challenged avoid trouble. In this article, take a math refresher course and possibly discover a few new ways of viewing complex equations and difficult hydraulic theory.

Square footage (sq. ft./ ft^2)

One of the most important and most used values in water math is square feet (ft^2) (see Figure 1). It determines filter loading rates (capacities), media backwash rates, pressure drop through a media bed and the starting calculation in establishing media volumes and bed depths (see Figure 2).

The equation for calculating the square feet within a circle is πr^2 (π = 3.14), where r is the radius or half the diameter (D). The industry sizes tanks and pipes in inches. Media and flowrate specifications come in square feet. Here is an easy shortcut to simplify a diameter expressed in inches and have the equation's result come out in square feet. The equation is:

$D^2/183$ or (D x D) ÷ 183. For further discussion, note that A ÷ B is mathematically represented as A over B or A ÷ B.

This equation replaces $\pi r^2 \div 144$, where r is inches and there are 144 square inches in a square foot. $D^2 \div 183$ ($D^2/183$) is accurate, quick and easy because manufacturers give their tank sizes in diameter inches.

Here is an example of the calculations for a 10-inch-diameter tank using both equations: Pi = π = 3.14. Pi is not a mysterious number. It is simply the number of times a circle's diameter goes around that circle's circumference.

D = 5; D^2 = 5 x 5 = 25

$\pi r^2 \div 144$ = 3.14 (25) /144; 78.5/144 = 0.54 ft^2 within a 10-inch circle or

$D^2 \div 183$ = (10 x10) ÷ 183 = 100/183 = 0.54 ft^2

Again, it is a quick and easy equation to use and remember ($D^2 \div 183 = D^2/183$). Try it. A 30-inch tank is 900/183 = 4.9 ft^2. It works every time.

A rectangle or square is simply sides A x B. If A = 2 ft. and B = 3 ft.; then the squared value is 2 x 3 = 2 (3) = 6. For further discussion, note that A x B is mathematically represented as A next to B or A(B) (see Figure 3).

Figure 1

Figure 2

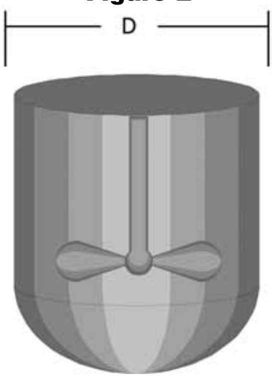


Figure 3

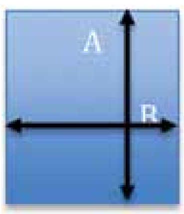
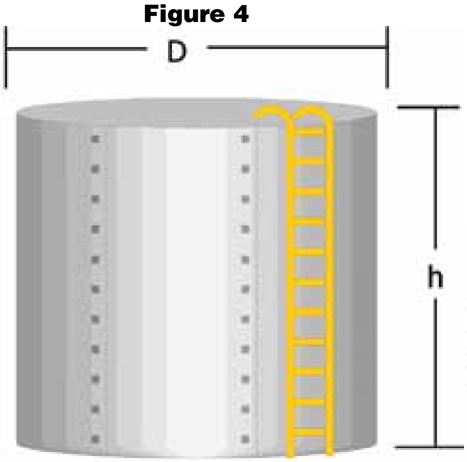


Figure 4



Volume (cubic feet) (cu. ft./ ft^3)

Volumes are three dimensional, requiring three measurements. If the square footage is known, simply multiply by the height and the result is cubic feet (see Figure 4). The geometric equation for calculating the cubic feet within a cylinder (tank) is: πr^2 h. Knowing the square footage of the cylinder, simply multiply ft^2 times the height. Looking at the 10-inch tank with 0.54 ft^2, then each foot of depth is 0.54 ft^3 (cubic feet). A three-foot bed depth in a 10-inch-diameter tanks is:

$D^2/183$ x 3 = 100/183 x 3 = 0.54 x 3 = 1.62 ft^3

Another example is a 24-inch-diameter tank with a three-foot bed depth:

$D^2/183$ x 3 = $D^2/183$ (3) = (24 x 24/183) (3) = 3.14 (3) = 9.42 ft^3

A rectangle is W x L x H. Use measurements in feet to get cubic feet.

Loading per square foot

Manufacturers provide hydraulic loading rates in their specifications under operating conditions (see Figure 5). Using a common loading rate of five gallons per

Figure 5

minute per square foot (5 gpm/ft^2), here is the math to determine the size of a media filter vessel to filter 20 gpm:

20 gpm ÷ 5 gpm/ft^2 = 20/5 = 4 ft^2

Knowing that particulate (i.e., iron, turbidity, etc.) loads at a rate of five gpm/ft^2 and the flow is 20 gpm, then the application requires 4 ft^2 of filter surface area. Using the available tank sizes (see Chart A), one can easily choose the tank required to handle the filtration of 20 gpm @ 5 gpm/ft^2. From the chart, it takes a 30-inch-diameter tank to handle 20 gpm at a hydraulic loading rate of 5 gpm/ft^2.

Here is a real-world application. Birm has a maximum service flowrate of 5 gpm/ft^2 (from manufacturer's conditions for operation). If one puts the proper amount of Birm in a 12-inch-diameter tank (30 to 36-inch bed depth) then the max flow through this filter is:

$D^2/183$ = (12 x 12) ÷ 183 = 144/184 = 0.79 ft^2;

5 gpm/ft^2 x 0.79 ft^2 = 3.95 gpm

Conditions for operation for Birm (Clack Corp.)

Alkalinity should be greater than two times the combined sulfate and chloride concentration.

Maximum water temp: 100°F/38°C

Water pH range: 6.8 to 9.0

Dissolved oxygen (DO) content must be equal to at least 15 percent of the iron (or iron and manganese) content.

Bed depth: 30 to 36 in.

Freeboard: 50 percent of bed depth (min.)

Backwash rate: 10-12 gpm/sq.ft.

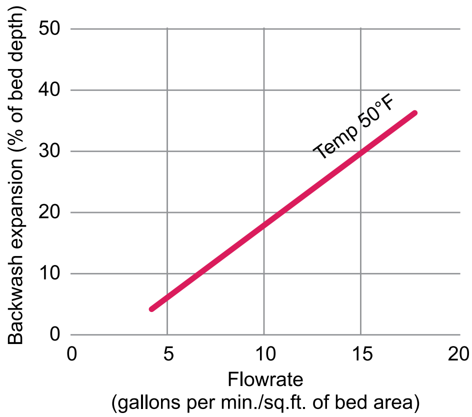
Backwash bed expansion: 20 to 40 percent of bed depth (min.)

Service flowrate: 3.5 to 5 gpm/sq.ft. intermittent flowrates and/or favorable local conditions may allow higher flowrates

12 x 0.79 = 12 (0.079) = 9.48 gpm

Because there is no such thing as a 9.48-gpm drain line flow control (DLFC), it makes sense to round out to a DLFC of 10 gpm. Just double-check the bed expansion curve to ensure that there is adequate freeboard to handle the bed expansion during backwash.

Figure 6. Backwash bed expansion



Backwash rate per square foot (gpm/ft^2)

Having the square foot calculation for a tank, one can easily calculate the backwash flowrate required for that tank. Assuming a backwash flowrate of 12 gpm/ft^2 and a 12-inch-diameter tank with 0.79 ft^2 (from Chart A), then the required flow to drain during backwash is:

Chart A

Tank diameter	D2/183 equation	Square ft. surface area	Cubic ft. per 2.5 ft. bed depth	Filter rate at 5 gpm/ft.	Backwash rate at 12 gpm/ft.
8	64/183	0.35	0.87	1.7	4.2
9	81/183	0.44	1.11	2.2	5.3
10	100/183	0.54	1.37	2.7	6.6
12	144/183	0.79	1.97	3.9	9.5
13	169/183	0.92	2.31	4.6	11.0
14	196/183	1.07	2.68	5.3	12.9
16	256/183	1.40	3.50	7.0	16.8
18	324/183	1.77	4.43	8.8	21.3
20	400/183	2.19	5.46	10.9	26.2
21	441/183	2.41	6.02	12.0	28.9
22	484/183	2.64	6.61	13.2	31.7
24	576/183	3.15	7.87	15.7	37.8
30	900/183	4.92	12.30	24.5	59.0
36	1296/183	7.08	17.70	35.4	85.0
42	1764/183	9.64	24.10	48.2	115.7
48	2304/183	12.59	31.48	62.9	151.1
54	2916/183	15.93	39.84	79.6	191.2
60	3600/183	19.67	49.18	98.3	236.1
63	3969/183	21.69	54.22	108.4	260.3

Bed expansion

Bed expansion (Figure 6) is a function of bed depth and the lift or expansion of the bed at a given flowrate to the drain during the backwash cycle. Filter beds require expansion during backwash to lift the captured material loaded on and in the top surface of the bed (see Figure 7). Allowing for a 30-percent specified bed expansion and 50°F water, looking at Figure 6, it specifies a 15 gpm/ft^2 flowrate. *Note:* Best practices look for 30 psi feed pressure at the given backwash flow for optimum results in residential applications. For large commercial and industrial systems, it typically requires 40 psi for adequate lift during backwash. Assuming a 30-inch bed depth, the bed will expand:

30 (30 percent) = 30 (0.3) = 9 inches

Knowing this, a 30-inch bed will expand to 39 inches during backwash. To ensure that the bed material remains in the tank

Figure 7

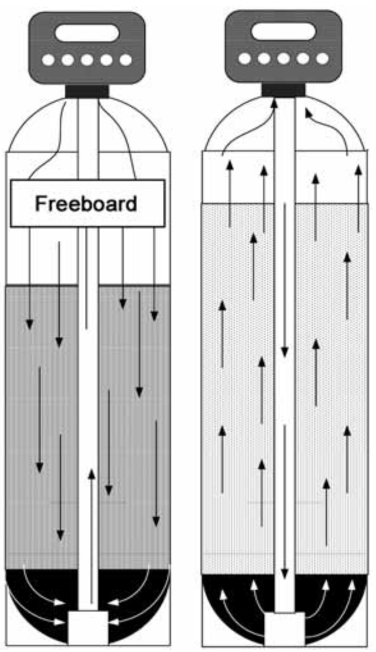
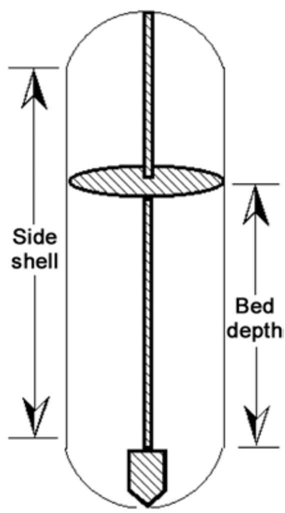


Figure 8



during backwash make sure that the freeboard (open space between the top of the media bed and the top of the tank) is greater than nine inches. It is common practice to leave 50-percent freeboard in media tanks. *Note:* Only use the side shell height (Figure 1) in calculating available freeboard and bed depth. If the media reaches the curvature of the tank during backwash, it is lost to the drain (see Figure 8).

Empty bed contact time (EBCT)

Empty bed contact time is a calculation used in adsorption, ion exchange and retention time. It is a calculation of how long water is in contact with media or chemicals. If an arsenic adsorptive media requires two minutes of EBCT, water must take two minutes to pass through the media. To calculate EBCT in measurements learned earlier in this lesson, use 7.48 gallons/ft^3. To calculate a 10-gpm flow with an EBCT of two minutes in cubic feet (ft^3), use these equations:

2-min. EBCT expressed in cubic feet is:

(7.48 gal/ft^3)/2 minutes = 7.48/2 = 3.74 gpm/ft^3

10 gpm with 2-minute EBCT is:

10 gpm/(3.74 gpm/ft^3) = 2.7 ft^3

Bed volumes (BV)

Bed volumes are a measurement of how much water can pass through a media bed before it reaches exhaustion—commonly called throughput. Staying with arsenic adsorption, a bed-life estimate for arsenic adsorptive media with 50 ppm As(V), ortho-phosphate 0.15 ppm, silica 20 ppm and a pH of 7.2 is 55,000 BV. One can calculate how many gallons will pass through the media before arsenic breaks through above the maximum contaminant level. Knowing that there are 7.48 gallons per cubic feet, one cubic foot of arsenic adsorptive media with a throughput of 55,000 BV will treat 7.48 gals/ft^3 (55,000 BV) = 441,400 gallons.

Playing with the numbers

Whenever an operating specification calls out a BV, maximum flowrate, EBCT, etc., it is providing the best-case scenario. An automobile engine is not designed to run at 8,000 rpm just because it can. The same engine runs optimally around 2,000 rpm. In the Olympics, the 100-meter dash is classified as a sprint. The 5,000-meter race is classified as long distance. This is because human beings cannot sprint for 5,000 meters. They will exhaust and fail.

Try to be conservative when looking at operating specifications and look for the optimum numbers, not the maximums. Manufacturers of POE and POU systems, media and related products provide conditions of operations. Refer to these specifications and use good old common sense when applying water technologies.

Sound science and engineering

Designing and troubleshooting equipment requires that one knows the math. Learning and using water math requires practice. Do the homework. Once the equations become part of everyday use, the numbers and calculations become instinctive—and make everyone smarter and better professionals.

About the author

♦ *Matthew Wirth, Layne Christensen Commercial Sales, Water Technologies POE/POU Division, is responsible for the region west of the Mississippi River. He is a 32-year professional in the water industry and an active trainer for several national organizations. Wirth has extensive experience in light C&I, POE and POU problem-water applications. A graduate of Concordia University in St. Paul, MN with a BA Degree in organizational management and communications, he received engineering training at the South Dakota School of Mines and Technology in Rapid City, SD. He can be reached at matthew.wirth@layne.com or cell phone, (319) 333-4174.*

About the company

♦ *Layne Water Technologies (www.laynewater.com) owns the LayneRT adsorptive technology and offers it in multiple residential and commercial configurations through a network of approved professionals. They can be contacted at (800) 216-5505.*

Addenda: Numbers Don't Lie – A Water Math Tutorial

1. In the **square footage section**, the author points out that square feet (ft^2) is a very common value in water math. The specific value the author is discussing is also known as the cross-sectional area of a media bed. The flowrate distributed over this area, reported most often by media manufacturers in gallons/minute/square foot (gpm/ft^2) is sometimes referred to as the surface flow.

2. Also in the **square footage section**, the illustration of the diameter and radius calculations for a 10 inch diameter tank, the value of 5 inches is incorrectly indicated as diameter. It is the radius, and as such, the first line of the calculation should look as follows:
   $r=5$ $r^2=5x5=25$

3. Including the units in a calculation can help verify that the calculation was set up correctly. When the units calculate to the correct value, the numbers will also.

   The conversion factor of 183 which helps to quickly convert from D^2 in square inches to the area of a circle (A) in square feet has the units of in^2/ft^2. If D = 10 in, then:

   $D^2 = 10 \text{ in} \times 10 \text{ in} = 100 \text{ in}^2$

   The formula for area, using the author's shortcut is:

   $\text{Area} = D^2/183$

   Adding the numbers and units we get:

   $\text{Area} = 100 \text{ in}^2/(183 \text{ in}^2/\text{ft}^2) = 0.54 \text{ ft}^2$

   Note: $1/(1/\text{ft}^2) = \text{ft}^2/1 = \text{ft}^2$

   When dividing by a fraction, the denominator (bottom number) of the fraction becomes the numerator (top number) of the answer.

4. In the **backwash rate per square foot section,** the author demonstrates how to determine the required backwash flowrate based on the media manufacturer's specification of 12 gpm/ft^2 and a known tank size, 12 inches in diameter. Chart A is a convenient reference for determining the cross-sectional area of the tank, which is 0.79 ft^2.

   Adding units to the sample calculation provided by the author helps verify that the calculation was set up correctly. Because we're looking for flowrate, the correct units would be gallons per minute (gpm). Using the dimensional analysis method to end up with only gpm, the calculation would be set up as follows:

Surface flowrate (gpm/$\cancel{ft^2}$) x Surface area ($\cancel{ft^2}$) = Backwash flowrate (gpm)

12 gpm/$\cancel{ft^2}$ x 0.79 $\cancel{ft^2}$ = 9.48 gpm

5. In the **empty bed contact time section,** the author introduces a conversion factor of 7.48 gallons/ft^3. Those familiar with EBCT calculations will recognize it as the conversion from volume in gallons to the volume in cubic feet.

6. Author's **updated bio** and **company information**
   Matthew Wirth is the Director of Training for North America at Canature WaterGroup. He is a 37-year professional in the water industry with experience in Industrial, Municipal, Commercial, POE, and POU water applications. He received his engineering training at the South Dakota School of Mines and Technology in Rapid City, SD and is a graduate of Concordia University in St. Paul, MN with a BA Degree in Organizational Management and Communications. He can be reached at matthew.wirth@canaturewg.com.

   Hydrotech, a division of Canature WaterGroup, is part of the fastest growing, most innovative manufacturer of high-quality water conditioning products in the world. Our 1.2 million sq. ft. ISO9001:2008 Quality Assurance Certified Facility features NSF Certified control valves, FRP tanks and assembled systems. With over 1000 years of industry experience, our team is dedicated to providing products with better features, quality and overall value – all backed by experienced, dedicated support.